ABBOTSLEIGH

AUGUST 2007

YEAR 12
ASSESSMENT 4
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Write using blue or black pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.

Total marks **(120)**

- Attempt Questions 1-10.
- All questions are of equal value.

Outcomes assessed

Preliminary course

P1 demonstrates confidence in using mathematics to obtain realistic solutions to problems.
P2 provides reasoning to support conclusions that are appropriate to the context
P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques
P5 understands the concept of a function and the relationship between a function and its graph
P6 relates the derivative of a function to the slope of its graph
P7 determines the derivative of a function through routine application of the rules of differentiation
P8 understands and uses the language and notation of calculus

HSC course

H1 seeks to apply mathematical techniques to problems in a wide range of practical contexts
H2 constructs arguments to prove and justify results
H3 manipulates algebraic expressions involving logarithmic and exponential functions
H4 expresses practical problems in mathematical terms based on simple given models
H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems
H6 uses the derivative to determine the features of the graph of a function
H7 uses the features of a graph to deduce information about the derivative
H8 uses techniques of integration to calculate areas and volumes
H9 communicates using mathematical language, notation, diagrams and graphs

Marks

Question 1 (12 marks)
Use a SEPARATE writing booklet

(a) Evaluate $\sqrt{\dfrac{85.7}{6.8\times 5.2}}$ correct to 2 significant figures. **2**

(b) Evaluate $\sum_{r=3}^{5} r^2$. **1**

(c) Solve $|3x-1|\geq 5$. **2**

(d) Rationalise the denominator of $\dfrac{2}{2\sqrt{3}+5}$. **2**

(e) Simplify $\dfrac{2}{b-3}\div\dfrac{6}{b^2-9}$. **2**

(f) Solve $\dfrac{2x-1}{3}-\dfrac{x-2}{2}=5$. **3**

Marks

Question 2 (12 marks)
Use a SEPARATE writing booklet

(a) Find the coordinates of the vertex of the parabola $(y-4)^2 = 8(x+3)$. **1**

(b) Differentiate with respect to x:

(i) $\dfrac{x^2}{x-1}$ **2**

(ii) $x\cos x$ **2**

(iii) $\log_e\left(x^3-1\right)$ **2**

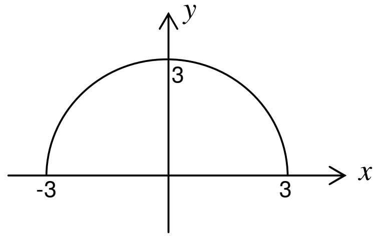
(c) Find $\int \dfrac{x^3-4}{x}\,dx$ **2**

(d) Evaluate $\int_0^3 6e^{2x}dx$ leaving your answer in exact form. **3**

Marks

Question 3 (12 marks)
Start a new booklet

(a)

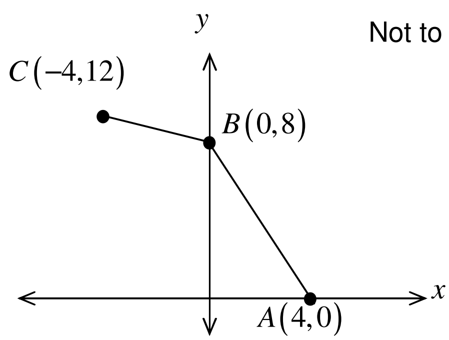


(i) State the equation of the semicircle shown above. **1**

(ii) State the domain of this function. **1**

(iii) State the range of this function. **1**

(b)



Not to Scale

$A(4,0)$, $B(0,8)$ and $C(-4,12)$ are 3 points on a number plane.

(i) Draw the diagram into your answer booklet and find the length of AB in simplest surd form. **2**

(ii) Find the gradient of the line AB. **1**

(iii) The line l is parallel to AB and passes through C. Show that the equation of l is $2x + y - 4 = 0$. **2**

(iv) Draw l on your diagram and find the coordinates of D, the point where l intersects the y axis. **1**

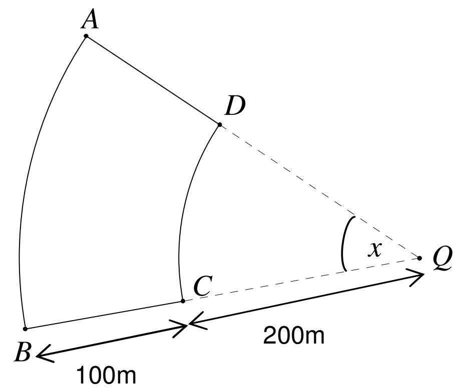
(v) Find the perpendicular distance from B to the line l. **2**

(vi) Find the area of parallelogram $ABCD$. **1**

Marks

Question 4 (12 marks)
Start a new booklet

(a)



In the figure AB and CD are circular arcs which subtend an angle x radians at the centre Q where $0 < x < \pi$ and AQ, BQ are radii. The length BC is 100m and CQ is 200m.

(i) Find expressions in terms of x for the length of the arcs AB and CD. **1**

(ii) A man lives at A and there is a bus stop at B, with the paths AB, BC, CD and DA in the figure forming a road system. For what values of x is it shorter for the man to walk along route $ADCB$ rather than along the arc AB? **2**

(iii) Given $x = \frac{\pi}{5}$ find the area enclosed by the roads linking A, B, C *and* D. **2**

(b) (i) Write down the discriminant of $x^2 - 2kx + 6k$. **2**

(ii) For what values of k is $x^2 - 2kx + 6k$ positive definite? **2**

(c) Find the volume of the solid of revolution formed by rotating the curve $y = x + \frac{1}{x}$ about the x axis between $x = 1$ and $x = 3$. **3**

Marks

Question 5 (12 marks)
Start a new booklet

(a) The fifth term of an arithmetic series is 14 and the sum of the first 10 terms is 165. Find the first term of the series. **3**

(b) Use Simpson's rule with 3 function values to estimate $\int_0^1 4^x \, dx$. **2**

(c) From the integers 1 to 25, one integer is chosen at random. What is the probability that it is:

(i) divisible by 5 and greater than 18. **1**

(ii) divisible by 5 or greater than 18. **1**

(d) The roots of the quadratic equation $px^2 - x + q = 0$ are -1 and 3. Find p and q. **2**

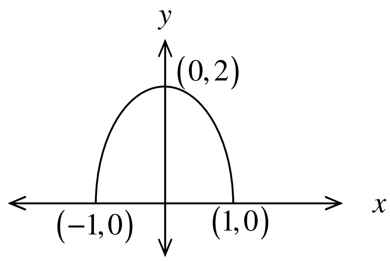
(e) The curve $y = ax^3 - 9x^2 + b$ has a minimum turning point at $(3, -12)$. Find the values of a and b. **3**

Marks

Question 6 (12 marks)
Start a new booklet

(a) Solve $\cos\theta = \dfrac{-\sqrt{3}}{2}$ for $0 \le \theta \le 2\pi$ **2**

(b)

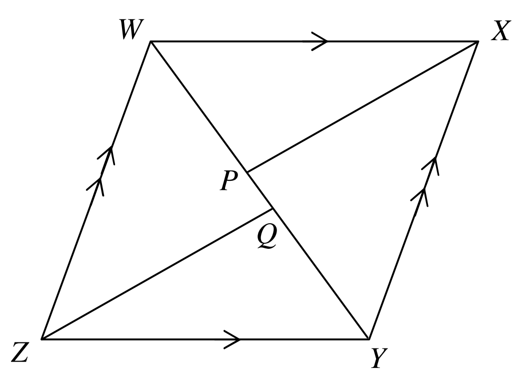


An ornamental arch window 2m wide and 2m high is to be made in the shape pictured on the axes pictured.

The architect decided that $y = 2\cos\dfrac{\pi}{2}x$ or $y = 2 - 2x^2$ would be suitable equations to use.

By finding the area of each possible window, determine which one would allow the greatest amount of light in. **4**

(c)



$WXYZ$ is a parallelogram.
XP bisects $\angle WXY$ and
ZQ bisects $\angle WZY$.
WY is a diagonal.

(i) Copy the diagram into your answer booklet and explain why $\angle WXY = \angle WZY$. **1**

(ii) Prove $\Delta WXP \equiv \Delta YZQ$. **3**

(iii) Hence find the length of PQ given $WY = 20\text{cm}$ and $QY = 8\text{cm}$. **2**

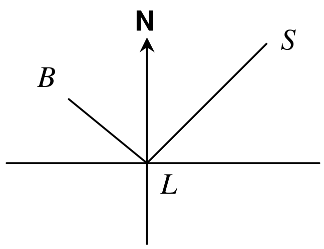
Marks

Question 7 (12 marks)
Start a new booklet

(a) (i) Sketch on the same number plane the graphs of $y = 3\sin 2x$ and $y = 1 - \cos x$ for $0 \le x \le 2\pi$. **3**

(ii) Using your graphs, determine the number of solutions the equation $3\sin 2x + \cos x = 1$ will have in the given domain. **1**

(b) From a lighthouse L a ship S bears $053°$ and is at a distance of 8 nautical miles. From L a boat B bears $293°$ and is at a distance of 6 nautical miles.

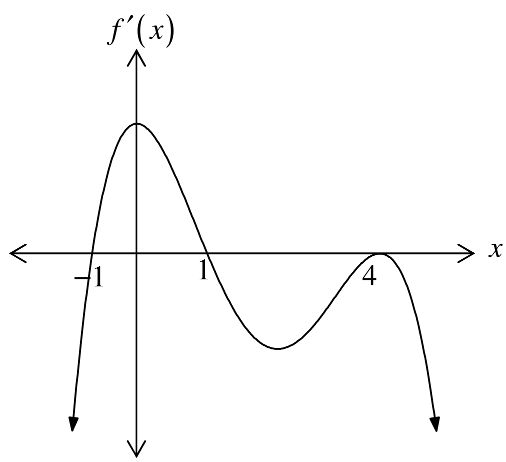


(i) Draw the diagram in your answer booklet and mark on the given information. **1**

(ii) Find the distance of ship S from Boat B. Give your answer as a surd. **2**

(iii) Find the bearing of ship S from Boat B. Give your answer to the nearest degree. **2**

(c)



Not to Scale

The graph of $y = f'(x)$ is sketched above.

(i) Write down the values of x where stationary points will occur in the graph of $y = f(x)$. **1**

(ii) Sketch $y = f(x)$ given that it passes through $(0,0)$ and $(4,-2)$. Clearly show any turning points or points of inflection. **2**

Marks

Question 8 (12 marks)
Start a new booklet

(a) If $y = e^{4x} + e^{2x}$

(i) Find $\dfrac{dy}{dx}$ **1**

(ii) Show that $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 8y = 0$ **2**

(b) The mass M in grams of a radioactive substance may be expressed as $M = Ae^{-kt}$ where t is the time in years and k is a constant.

(i) If initially the mass is 10 grams, find A. **1**

(ii) After 5 years the mass in 9 grams. Find the mass after 20 years. **2**

(c) A gold coin is biased so that the probability of tossing a head is $\dfrac{2}{3}$. A silver coin is 'fair' with an equal chance of tossing a head or a tail.

(i) The two coins are tossed together. what is the probability of tossing

(α) 2 heads. **1**

(β) a head and a tail. **2**

(γ) at least 1 tail. **1**

(ii) One of the coins is selected at random and then tossed twice. What is the probability of tossing 2 tails. **2**

Marks

Question 9 (12 marks)
Start a new booklet

(a) The line $y = mx$ is a tangent to the curve $y = e^{3x}$. Find the value of m. **2**

(b) Mary decided to contribute to a superannuation fund by investing \$3000 each year at the beginning of 1990. The interest was paid at a rate of 7% p.a. for 10 years after which it was changed to 8% p.a. starting January 1, 2000. She plans to continue to invest in this fund until the year 2010.

(i) Show that after 10 years her investment is worth \$44 350.80. **2**

(ii) How much is her investment worth at the end of the 11th year? **1**

(iii) How much is her investment worth at the end of 2010? **2**

(c) The cost of running a long distance transport truck is $\$\left(\dfrac{V^2}{3} + 600\right)$ **<u>PER HOUR</u>** where V is the speed in kilometres per hour.

(i) Find an expression for the number of hours travelled over a distance of 100 km. **1**

(ii) Show that the <u>Total</u> Cost of running the truck for 100km is given by

$$C = \frac{100V}{3} + \frac{60000}{V}$$ **1**

(iii) Using the expression in part (ii) find the most economical speed for running the truck for 100 km at that speed. (i.e. the speed which gives the minimum cost) **3**

Marks

Question 10 (12 marks)
Start a new booklet

(a) (i) For what values of x does the geometric series $1+4x^2+16x^4+64x^6+...$ have a limiting sum? **2**

(ii) Find the limiting sum when $x=\frac{1}{3}$. **2**

(b) A rainwater tank with a volume of 9m^3 is installed in a new house. At 8am rain begins to fall and flows into the empty tank at the rate given by

$$\frac{dV}{dt}=\frac{36t}{t^2+20}$$

where t is the time in hours and V is the volume measured in cubic metres ($t=0$ is represented by 8am).

(i) Show that the volume of water in the tank at time t is given by $V=18\log_e\left(\frac{t^2+20}{20}\right), \quad t>0$. **2**

(ii) Find the time when the tank will be completely filled with water (to the nearest minute). **3**

(iii) Later, when the tank is full and the rain has stopped, Louise turns on the pump which pumps the water out at the rate given by $\frac{dV}{dt}=\frac{t^2}{k}$. The pump continues for 5 hours until the tank is empty. Find the value of k. **3**

END OF PAPER

2007 Mathematics Trial Solutions

1 (a) $\sqrt{\dfrac{85\cdot 7}{6\cdot 8\times 5\cdot 2}} = 1\cdot 55680$

$= \underline{\underline{1\cdot 6}}$ to 2 sign figs

(b) $\sum_{r=3}^{5} r^2 = 3^2+4^2+5^2$

$\underline{\underline{= 50}}$

(c) $3x-1 \leqslant -5$ or $3x-1 \geqslant 5$

$3x \leqslant -4$ $\quad$ $3x \geqslant 6$

$\underline{\underline{x \leqslant -\frac{4}{3} \text{ or } x \geqslant 2}}$

(d) $\dfrac{2}{2\sqrt{3}+5} \times \dfrac{2\sqrt{3}-5}{2\sqrt{3}-5}$

$= \dfrac{4\sqrt{3}-10}{12-25} = \underline{\underline{\dfrac{4\sqrt{3}-10}{-13}}} = \underline{\underline{\dfrac{10-4\sqrt{3}}{13}}}$

(e) $\dfrac{2}{b-3} \div \dfrac{6}{b^2-9} = \dfrac{2}{b-3} \times \dfrac{b^2-9}{6}$

$= \dfrac{2}{b-3} \times \dfrac{(b+3)(b-3)}{6}$

$= \underline{\underline{\dfrac{b+3}{3}}}$

(f) $\dfrac{2x-1}{3} - \dfrac{x-2}{2} = 5$

$2(2x-1) - 3(x-2) = 30$

$4x-2-3x+6 = 30$

$x+4 = 30$

$\underline{\underline{x = 26}}$

2 (a) $(y-4)^2 = 8(x+3)$ has vertex $\underline{\underline{(-3,4)}}$

(b) (i) $\dfrac{d}{dx}\left(\dfrac{x^2}{x-1}\right) = \dfrac{(x-1)2x - x^2}{(x-1)^2}$

$= \dfrac{2x^2-2x-x^2}{(x-1)^2}$

$= \underline{\underline{\dfrac{x^2-2x}{(x-1)^2}}}$

(ii) $\dfrac{d}{dx}(x\cos x) = x \times -\sin x + \cos x$

$= \underline{\underline{\cos x - x\sin x}}$

(iii) $\dfrac{d}{dx}\left(\log_e(x^3-1)\right) = \underline{\underline{\dfrac{3x^2}{x^3-1}}}$

(c) $\displaystyle\int \frac{x^3-4}{x}\,dx = \int \frac{x^3}{x} - \frac{4}{x}\,dx$

$= \displaystyle\int x^2 - \frac{4}{x}\,dx$

$= \underline{\underline{\dfrac{x^3}{3} - 4\log_e x}}$

(d) $\displaystyle\int_0^3 6e^{2x}\,dx = \left[\frac{6e^{2x}}{2}\right]_0^3$

$= 3(e^6 - e^0)$

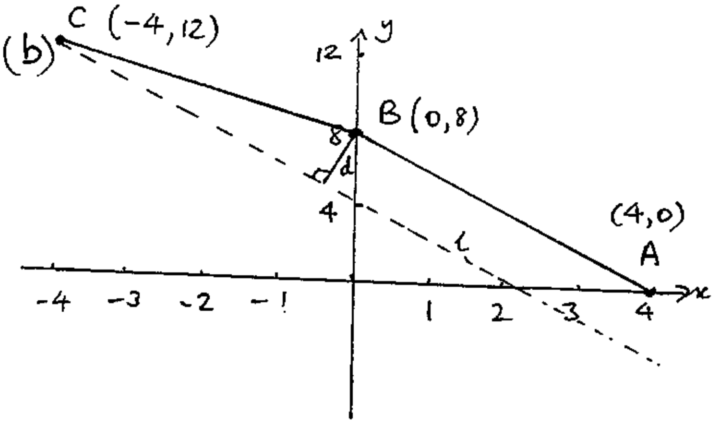
$= \underline{\underline{3(e^6-1)}}$

③ (a) (i) $y = \sqrt{9-x^2}$ is eqn of semicircle

(ii) Domain : $-3 \leq x \leq 3$

(iii) Range : $0 \leq y \leq 3$

(b)



(i) $AB = \sqrt{4^2+8^2}$ (By Pythagoras or distance formula)

$= \sqrt{80}$

$= 4\sqrt{5}$

(ii) $m = \frac{0-8}{4-0} = -2$

(iii) $y - y_1 = m(x - x_1)$

$y - 12 = -2(x+4)$

$y - 12 = -2x - 8$

$2x + y - 4 = 0$

(iv) When $x = 0$ $\quad 0 + y - 4 = 0$

$y = 4$

$D(0,4)$

(v) $d = \frac{|2x+y-4|}{\sqrt{2^2+1^2}}$

$= \frac{|2\times 0 + 8 - 4|}{\sqrt{5}}$

$= \frac{4}{\sqrt{5}}$ units

(vi) $A = bh$

$= AB \times d$

$= 4\sqrt{5} \times \frac{4}{\sqrt{5}}$

$= 16$ square units

④ (a) (i) $l = r\theta$

$AB = 300x$

$CD = 200x$

(ii) Route ADCB $= 100 + 200x + 100$

$= 200 + 200x$

$200 + 200x < 300x$

$200 < 100x \quad \therefore x > 2$

$2 < x < \pi$ radians (since $0 < x <$ … by defn).

(iii) Area ADCB = Area sector AQB − Area sector DQ…

$= \frac{1}{2} \times 300^2 \times \left(\frac{\pi}{5}\right) - \frac{1}{2} \times 200^2 \times \frac{\pi}{5}$

$= \frac{\pi}{10}(90000 - 40000)$

$= 5000\pi$ square units

(b) (i) $x^2 - 2kx + 6k$

$\Delta = b^2 - 4ac$

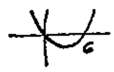
$= (-2k)^2 - 4 \times 1 \times 6k$

$= 4k^2 - 24k$

(ii) positive definite means $\Delta < 0$

$4k^2 - 24k < 0$

$4k(k-6) < 0$



$0 < k < 6$

(c) $V = \pi \int_1^3 y^2\, dx$

$= \pi \int_1^3 \left(x + \frac{1}{x}\right)^2 dx$

$= \pi \int_1^3 x^2 + 2 + \frac{1}{x^2}\, dx$

$= \pi \int_1^3 x^2 + 2 + x^{-2}\, dx$

$= \pi \left[\frac{x^3}{3} + 2x - x^{-1}\right]_1^3$

$= \pi \left[\left(\frac{27}{3} + 6 - \frac{1}{3}\right) - \left(\frac{1}{3} + 2 - 1\right)\right]$

$= \pi \left(\frac{44}{3} - \frac{4}{3}\right)$

$= \frac{40\pi}{3}$ cubic units

5 (a) $T_5 = 14$

$S_{10} = 165$

$a + 4d = 14$ (1)

$\frac{10}{2}(2a + 9d) = 165$ (2)

$5(2a + 9d) = 165$

$2a + 9d = 33$ (3)

(1) x 2: $2a + 8d = 28$ (4)

(3) − (4) $d = 5$

sub in (1) $a + 4 \times 5 = 14$

$\underline{\underline{a = -6}}$

(b)

x	0	$\frac{1}{2}$	1
4^x	1	2	4

$\int_0^1 4^x \, dx = \frac{1-0}{6}(1 + 4 \times 2 + 4)$

$= \frac{1}{6} \times 13$

$= \underline{\underline{\frac{13}{6}}}$

(c)(i) ÷ by 5 and > 18 is 20, 25

∴ Prob = $\underline{\underline{\frac{2}{25}}}$

(ii) ÷ by 5 or > 18 5, 10, 15, 20, 25, 19, 21, 22, 23, 24

∴ Prob = $\underline{\underline{\frac{10}{25}}} = \underline{\underline{\frac{2}{5}}}$

(d) $px^2 - x + q = 0$ roots −1, 3

sum of roots $= -\frac{b}{a} = \frac{1}{p}$

$-1 + 3 = \frac{1}{p}$ ∴ $2 = \frac{1}{p}$ $\underline{\underline{p = \frac{1}{2}}}$

product of roots $-1 \times 3 = \frac{q}{p}$

$\frac{1}{2} \times -3 = q$ $\underline{\underline{q = -\frac{3}{2}}}$

(e) $y = ax^3 - 9x^2 + b$

$y' = 3ax^2 - 18x$

When $x = 3$ $y' = 0$ $0 = 27a - 54$

$27a = 54$

$\underline{\underline{a = 2}}$

$y = 2x^3 - 9x^2 + b$

sub in $x = 3$ $y = -12$ (lies on curve)

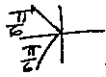
$-12 = 2 \times 27 - 9 \times 9 + b$

$-12 = 54 - 81 + b$

$-12 = -27 + b$ $\underline{\underline{b = 15}}$

6 (a) $\cos\theta = -\frac{\sqrt{3}}{2}$ $0 \le \theta \le 2\pi$

Basic angle $= \frac{\pi}{6}$



$\theta = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\underline{\underline{\theta = \frac{5\pi}{6}, \frac{7\pi}{6}}}$

(b) $2\int_0^1 2\cos\frac{\pi}{2}x \, dx$

$= 4\int_0^1 \cos\frac{\pi}{2}x \, dx = 4\left[\frac{2\sin\frac{\pi}{2}x}{\pi}\right]_0^1$

$= \frac{8}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right)$

$= \underline{\underline{\frac{8}{\pi}}} \doteq 2.5464$ square units

$2\int_0^1 2 - 2x^2 \, dx = 4\int_0^1 1 - x^2 \, dx$

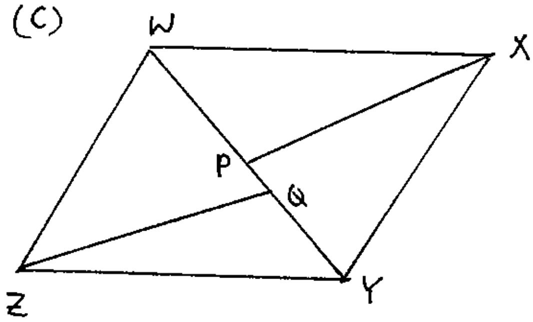
$= 4\left[x - \frac{x^3}{3}\right]_0^1$

$= 4\left[\left(1 - \frac{1}{3}\right) - (0 - 0)\right]$

$= \frac{8}{3} = \underline{\underline{2\frac{2}{3}}} = 2.\dot{6}$

The parabolic equation would let more light in.

(c)



(i) ∠WXY = ∠WZY (opposite angles of a parallelogram are equal)

(ii) In △WXP, △YZQ

∠WXP = $\frac{1}{2}$∠WXY (XP bisects ∠WXY)

∠QZY = $\frac{1}{2}$∠WZY (ZQ bisects ∠WZ

since ∠WXY = ∠WZY (from (i)) ∠WXP = ∠Q

WX = ZY (opposite sides of parallelogr

∠XWY = ∠ZYW (alternate angles, WX ∥ ZY)

∴ △WXP ≡ △YZQ (AAS)

(iii) Since QY = 8, WP = 8 (correspondi sides in congruent triangles)

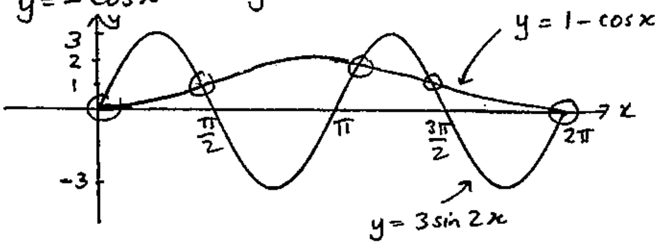
so PQ = 20 − (8 + 8)

$\underline{\underline{PQ = 4 \text{ cm}}}$

7 (a) (i) $y = 3\sin 2x$ amplitude 3 period π

$y = 1 - \cos x$ amplitude 1 Period 2π

$y = -\cos x$ $y = 1 - \cos x$ shifted up 1



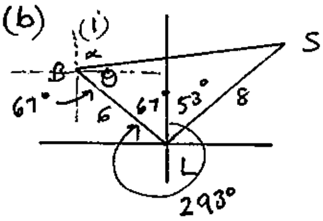
(ii) $3\sin 2x + \cos x = 1$

$3\sin 2x = 1 - \cos x$

Solution will be the number of points of intersection between the graphs $y = 3\sin 2x$ and $y = 1 - \cos x$

There are 5 solutions for $0 \le x \le 2\pi$

(b)



$BS^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 120°$

$BS^2 = 148$

$BS = \sqrt{148}$

(iii) Need to find θ

$\frac{\sin\theta}{8} = \frac{\sin 120°}{\sqrt{148}}$

$\sin\theta = \frac{8\sin 120°}{\sqrt{148}}$

$= 0.56949$

$\theta = 34° 43' = 35°$

$\alpha = 180 - 35° - 67°$

$= 78°$

$\therefore$ Bearing is 078°

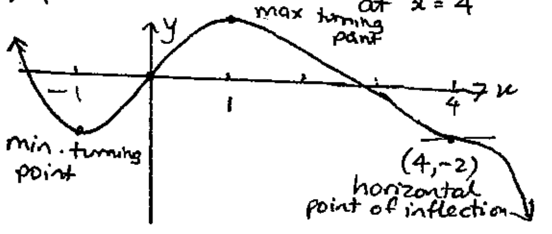
(c) (i) stat pts occur when $f'(x) = 0$

ie. when $x = -1, 1, 4$

(ii) As $x \to -1^-$ $f'(x) < 0$
$x \to -1^+$ $f'(x) > 0$ $\therefore$ minimum at $x = -1$

As $x \to 1^-$ $f'(x) > 0$
$x \to 1^+$ $f'(x) < 0$ $\therefore$ maximum at $x = 1$

As $x \to 4^-$ $f'(x) < 0$
$x \to 4^+$ $f'(x) < 0$ $\therefore$ horizontal point of inflection at $x = 4$



8 (a) (i) $y = e^{4x} + e^{2x}$

$\frac{dy}{dx} = 4e^{4x} + 2e^{2x}$

(ii) $\frac{d^2y}{dx^2} = 16e^{4x} + 4e^{2x}$

$LHS = \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y$

$= 16e^{4x} + 4e^{2x} - 6(4e^{4x} + 2e^{2x}) + 8(e^{4x} + e^{2x})$

$= 16e^{4x} + 4e^{2x} - 24e^{4x} - 12e^{2x} + 8e^{4x} + 8e^{2x}$

$= 0$

$= RHS.$

(b) (i) $M = Ae^{-kt}$

$t = 0$ $M = 10$ $10 = Ae^0$

$\therefore A = 10$

(ii) $t = 5$ $M = 9$

$9 = 10e^{-k \times 5}$

$\frac{9}{10} = e^{-5k}$

$\ln\frac{9}{10} = -5k$

$k = -\frac{1}{5}\ln\left(\frac{9}{10}\right) \doteqdot 0.0210721$

When $t = 20$ $M = 10e^{-20 \times 0.0210721}$

$= 6.561$

mass is 6.561 grams

(c) (i) (α) $P(\overset{G}{H}\overset{S}{H}) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

(β) $P(\overset{G}{H}\overset{S}{T}) + P(\overset{G}{T}\overset{S}{H}) = \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{3} + \frac{1}{6}$

$= \frac{1}{2}$

(γ) P(at least 1 tail) $= 1 - P(HH)$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

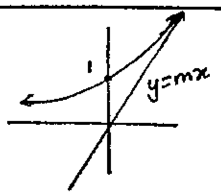
(ii) P(gold coin, T, T) + P(silver coin, T, T)

$= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{18} + \frac{1}{8}$

$= \frac{13}{72}$

9 (a)



$y = e^{3x}$

$\frac{dy}{dx} = 3e^{3x}$

gradient of $y = mx$ is m

so $m = 3e^{3x}$

line + curve intersect when $mx = e^{3x}$

But $m = 3e^{3x}$ so $3e^{3x}x = e^{3x}$

($\div$ by e^{3x}) $3x = 1$

$x = \frac{1}{3}$

$\therefore$ intersect at $x = \frac{1}{3}$

when $x = \frac{1}{3}$ $m = 3e^{3\times\frac{1}{3}}$

$\underline{m = 3e}$

(b)(i) end 1990 1st 3000 amounts to $3000(1.07)^1$

1991 2nd 3000 " " $3000(1.07)^2$

⋮

1999 10th 3000 " " $3000(1.07)^{10}$

geometric series $3000(1.07)^1 + 3000(1.07)^2 + \dots (1.07)^{10}$

Total $= \frac{a(r^n - 1)}{r - 1} = \frac{3000(1.07)(1.07^{10} - 1)}{1.07 - 1}$

$= \underline{\$44\,350.80}$

(ii) At end of 11th year

Amt $= 44\,350.80\,(1.08) + 3000(1.08)$

$\underline{= \$\,51\,138.86}$

(or $47\,350.8 \times 1.08$)

(iii) \$44 350.80 earns 8% compound interest for 11 years (2000 begin – 2010 end)

\$3000 invested every year at 8% for 11 years

Total $= 44\,350.80\,(1.08)^{11} + \frac{3000(1.08)(1.08^{11} - 1)}{1.08 - 1}$

$= 103\,410.05 + 53\,931.38$

$\underline{= \$157\,341.43}$

(c) (i) Speed $= \frac{\text{Dist}}{\text{Time}}$ so Time $= \frac{D}{S}$

$\underline{= \frac{100}{V}}$

(ii) Total cost = cost/hr × no. of hours

$= \left(\frac{V^2}{3} + 600\right) \times \frac{100}{V}$

$C = \frac{100V}{3} + \frac{60000}{V}$

(iii) $\frac{dC}{dV} = \frac{100}{3} - 60000V^{-2}$

Let $\frac{dC}{dV} = 0$ $\frac{100}{3} - \frac{60000}{V^2} = 0$

$\frac{100}{3} = \frac{60000}{V^2}$

$V^2 = \frac{180000}{100} = 1800$

$V = \sqrt{1800} = 30\sqrt{2}$ km/hr (take positive)

$\frac{d^2C}{dV^2} = +\frac{120000}{V^3}$

when $V = 30\sqrt{2}$ $\frac{d^2C}{dV^2} = \frac{120\,000}{(30\sqrt{2})^3} > 0$ ∪

$\therefore$ $\underline{V = 30\sqrt{2}}$ gives the minimum cost

$\underline{V \doteqdot 42.4 \text{ km/hr}}$

10 (a) (i) Limiting sum when $-1 < r < 1$

$1 + 4x^2 + 16x^4 + 64x^6 + \dots$

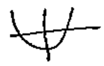
$r = 4x^2$

so $-1 < 4x^2 < 1$

$4x^2$ is always positive so just need to solve $4x^2 < 1$

$x^2 < \frac{1}{4}$

$x^2 - \frac{1}{4} < 0$



$(x + \frac{1}{2})(x - \frac{1}{2}) < 0$

$\underline{-\frac{1}{2} < x < \frac{1}{2}}$

(ii) when $x = \frac{1}{3}$

$4x^2 = \frac{4}{9}$

$S_\infty = \frac{a}{1 - r}$

$= \frac{1}{1 -}$

$= \underline{\frac{9}{5}}$

(b)(i) $\frac{dV}{dt} = \frac{36t}{t^2 + 20}$

$V = \int \frac{36t}{t^2 + 20}\,dt$

$= 18 \int \frac{2t}{t^2 + 20}\,dt$

$V = 18\log_e(t^2 + 20) + C$

$t = 0$ $V = 0$ $0 = 18\log_e(0 + 20) + C$

$C = -18\log_e(20)$

$V = 18\log_e(t^2 + 20) - 18\log_e 20$

$\underline{= 18\log_e\left(\frac{t^2 + 20}{20}\right)}$

(ii) Tank will be filled when $V = 9$

$9 = 18\log_e\left(\frac{t^2 + 20}{20}\right)$

$\frac{1}{2} = \log_e\left(\frac{t^2 + 20}{20}\right)$

$\frac{t^2 + 20}{20} = e^{\frac{1}{2}}$

$t^2 = 20e^{\frac{1}{2}} - 20$

$t^2 = 12.9744$ $t = \sqrt{12.9744}$ take +ve fo[r] time

$t = 3.6$ hours

$= 3$ hrs 36 mins

(iii) $V = \int \frac{t^2}{k}\,dt$

$V = \frac{t^3}{3k} + C$

$t = 0$ $V = 9$ $9 = 0 + C$

$V = \frac{t^3}{3k} + 9$

But when $t = 5$ $V = 0$

$0 = \frac{5^3}{3k} + 9$

$-9 = \frac{125}{3k}$

$k = -\frac{125}{27}$

or

$V = \int_0^5 \frac{t^2}{k}\,dt$

$9 = \left[\frac{t^3}{3k}\right]_5^0$

$9 = \left(\frac{125}{3k} - 0\right)$

$3k = \frac{125}{9}$

$k = \frac{125}{9}$

emptying so ma[ke] $-125/9$